I. Solution by the PROPOSER.

For each pair of lines a second pair may be drawn in opposite directions, dividing the surface of the circle into four portions each of which is included between two of the lines and the circumference. Hence the whole number of surfaces thus cut off may be arranged in sets of four such that the areas of each set shall equal the area of the circle. Hence the average required is $\frac{1}{4}a^2\pi$, where a is the radius of the circle.

Query I. As problem 32 does not describe how the lines are to be drawn to form the "sector" this is a particular case of that problem.

Query II. This query was proposed for information. Some one may be able to give authority for the use of the word in this sense. It is contrary to the usual definition.

Query III. It is the opinion of the writer that the use of the word *random* in average problems is the result of confusion of ideas, and although sometimes convenient is never correct.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let P be the given point. Through P draw the two chords MN, SR dividing the surface of the circle into the four surfaces A, B, C, D.

Then $A+B+C+D=\pi r^2$.

Since P can be taken anywhere on the surface of the circle and the lines MN, SR can make any angle from 0 to π, the average area of A=average area of B=average area of C=average area of D.

$\therefore$ $A=B=C=D=\frac{1}{4}\pi r^2$.

After carefully examining problem 32 I am inclined to think the above result the true answer to that problem also.

59. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A circle is rolling along a horizontal straight line. The uniform velocity of the center is v. Find the average velocity of a point of the circumference.

Solution by JOHN M. COLAW, A. M., Monterey, Va.; JOSIAH H. DRUMMOND, LL. D., Portland, Me.; M. E. GRABER, Mt. Vernon, O.; and the PROPOSER.

For the cycloid traced by the point, we have

$$\left.\begin{array}{l} x=a\theta-a\sin\theta \\ y=a-a\cos\theta \end{array}\right\},$$

$$dx=a(1-\cos\theta)d\theta\,;\quad dy=a\sin\theta d\theta.$$

$\therefore$ $ds^2=dx^2+dy^2, =2a^2d\theta^2(1-\cos\theta), =2a^2d\theta^2(2\sin^2\tfrac{1}{2}\theta)$.

$\therefore$ $ds=2a\sin\tfrac{1}{2}\theta d\theta$.

Now $OT=vt=a\theta$. $\therefore$ $dt=(a/v)d\theta$.

$\therefore$ $ds/dt=2a\sin\tfrac{1}{2}\theta d\theta\div(a/v)d\theta, =2v\sin\tfrac{1}{2}\theta$, the variable velocity of P.

$$\therefore \text{ the required average} = \frac{2v\int_0^\pi \sin\frac{1}{2}\theta d\theta}{\int_0^\pi d\theta} = 4v/\pi.$$

Also solved by *G. B. M. ZERR.*

MISCELLANEOUS.

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56. Proposed by S. HART WRIGHT, A. M., M. D., Ph. D., Penn Yan, N. Y.

In latitude 40° N.$=\lambda$, when the moon's declination is 5° 23′ N.$=\delta$, and the sun's declination 9° 52′ S.$=-\delta'$, how long after sunset will the cusps of the moon's crescent set synchronously, the moon having recently passed its conjunction with the sun ?

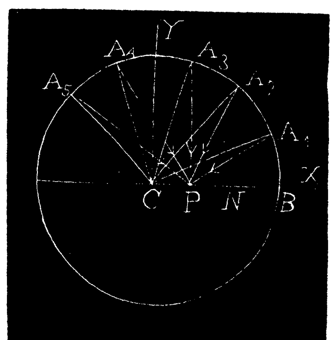
[NOTE. Problem 56 is identical with problem 54, and need not be here reproduced. See January number, pages 27 and 28, for two solutions. EDIROR.]

57. Proposed by GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.

A particle is placed very near the center of a circle, round the circumference of which n equal repulsive forces are symmetrically arranged; each force varies inversely as the mth power of its distance from the particle. Show that the resultant force is approximately $\dfrac{m_1 n(m-1)}{2r^{m+1}}\times CP$, and tends to the center of the circle, where m_1 is the mass of the particle, CP its distance from the center of the circle, and r the radius of the circle.

I. Solution by the PROPOSER.

Let the particle be at P, and C the center of the circle. Suppose the forces to be at A_1, A_2, ... $CP=x$, and $\angle A_2CB=\theta$. Then $\angle A_1CA_2=\angle A_2CA_3$ $=\ldots\ldots=360°/n=\beta$, say. Draw A_2N at right angles to CB. Consider the force at A_2. Then,



$$X=[m_1/(A_2P)^m]\cos A_2PN=[m_1(r\cos\theta-x)] /(r^2+x^2-2rx\cos\theta)^{\frac{1}{2}(m+1)}.$$

Let $m_1/r^{\frac{1}{2}(m+1)}=M$. Then, since x is small, neglecting terms containing higher powers of x than the first, we have

$$X=M\left[r^{\frac{1}{2}(1-m)}\cos\theta+\frac{\cos2\theta.x.r^{-\frac{1}{2}(m+1)}}{2} + \tfrac{1}{2}(m-1)xr^{-\frac{1}{2}(m+1)}\right].$$

To obtain the total force on P along X take the sum of n such expressions for all values of θ from $\theta=\alpha$ to $\theta=\alpha+(n-1)\beta$. Hence,